

CXV. On Sir J. J. Thomson's Model of a Light-Quantum.
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IT is commonly accepted that the quantum theory is almost incompatible with Maxwell's equations ("Clerk Maxwell's Electromagnetic Theory," by H. A. Lorentz, the Rede Lecture, 1923, p. 34); however, Sir J. J. Thomson† constructed a most valuable model of a light-quantum which, as will be shown, is formally consistent with the system of Maxwell's equations.

By the way of examining the simplest case of a light-quantum, we shall explain the difference of the Faraday-Maxwell-J. J. Thomson's electrodynamics from the usual Maxwell-Hertz-Lorentz's interpretation of an electromagnetic field, the form of the differential equations for the free æther being the same in both theories.

The light-quantum in the model of Sir J. J. Thomson consists in a ring-shaped Faraday tube moving with the speed of light at right angles to its own plane. The ring is preceded and followed by ordinary waves of feeble intensity; their wave-length is equal to the circumference of the ring. The energy contained in such a ring will be of the form $h\nu$ (Planck's law) if we admit, with Sir J. J. Thomson, that the cross-section of the tube is proportional to the square of the radius of the ring.

These two fundamental laws, as will be shown, are formally in accordance with the system of Maxwell's equations under the condition that we accept Sir J. J. Thomson's interpretation of an electromagnetic field.

Let E_α (fig. 1) be a closed ring-shaped Faraday tube referred to fixed axes. Let us suppose that at $t=0$ the plane of the ring coincides with the plane ZOY, and that u_x is the velocity of the ring. The Maxwell's equations for the free æther are of the form

$$\operatorname{div} \mathbf{E} = 0, \quad (1)$$

$$\operatorname{curl} \mathbf{M} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (2)$$

$$\operatorname{div} \mathbf{M} = 0, \quad (3)$$

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{M}}{\partial t}, \quad (4)$$

* Communicated by the Author.

† "A Suggestion as to the Structure of Light," Phil. Mag. [6] xlviii. p. 737 (1924); "The Structure of Light," Phil. Mag. l. p. 1182 (1925).

where c is the velocity of light, \mathbf{E} the electric and \mathbf{M} the magnetic intensity. In the case of stationary motion the operation $\frac{1}{c} \frac{\partial}{\partial t}$ will be replaced by $-\beta_x \frac{\partial}{\partial x}$ if $\beta_x = \frac{u_x}{c}$, and the equations (2) and (4) will be transformed into

$$\operatorname{curl} \bar{\mathbf{M}} = 0 \quad (2')$$

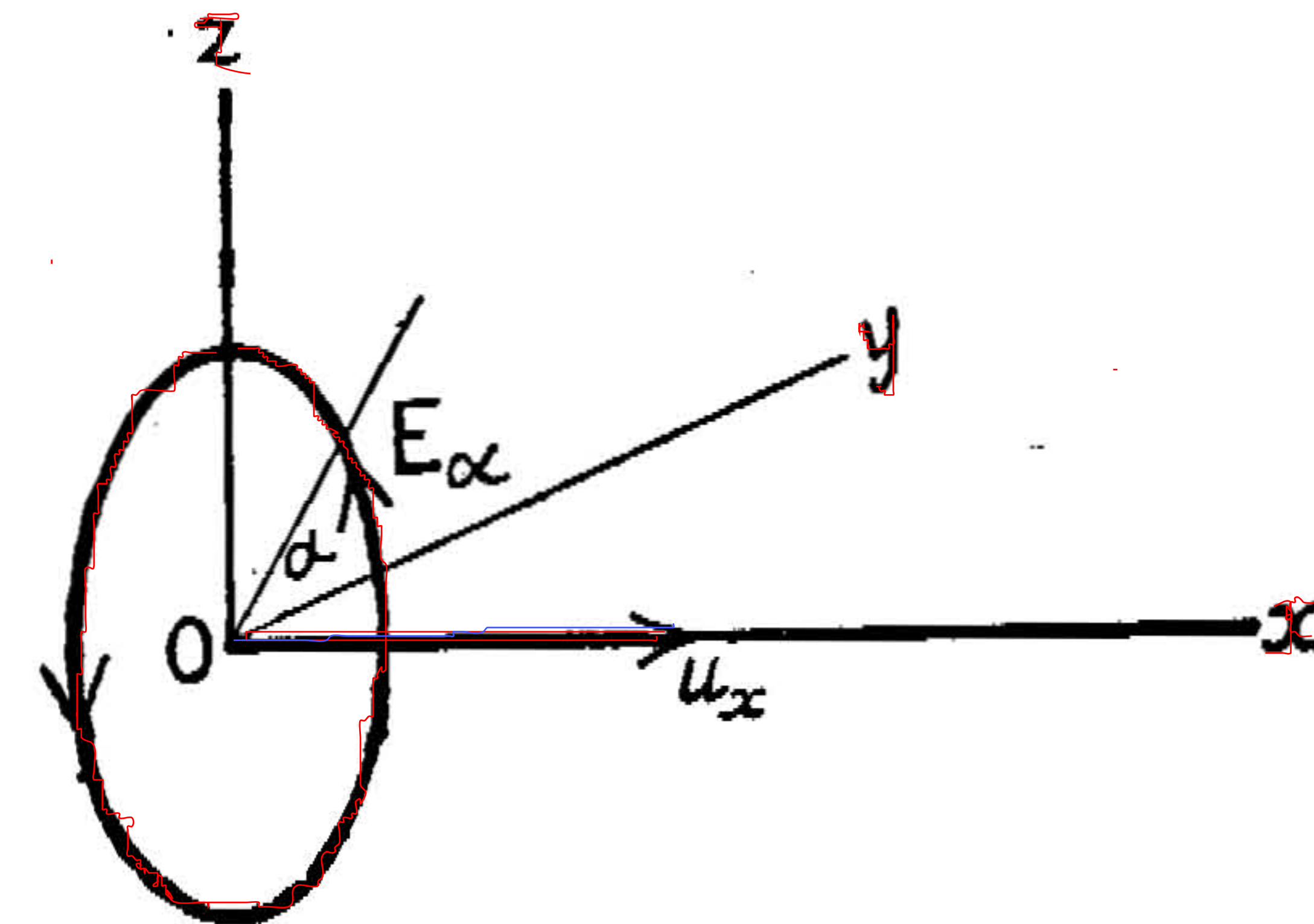
$$\text{and} \quad \operatorname{curl} \bar{\mathbf{E}} = 0, \quad (4')$$

where $\bar{\mathbf{M}}$ has for components

$$M_x, M_y + \beta_x E_z, M_z - \beta_x E_y$$

$$\text{and} \quad \bar{\mathbf{E}} \dots E_x, E_y - \beta_x M_z, E_z + \beta_x M_y.$$

Fig. 1.



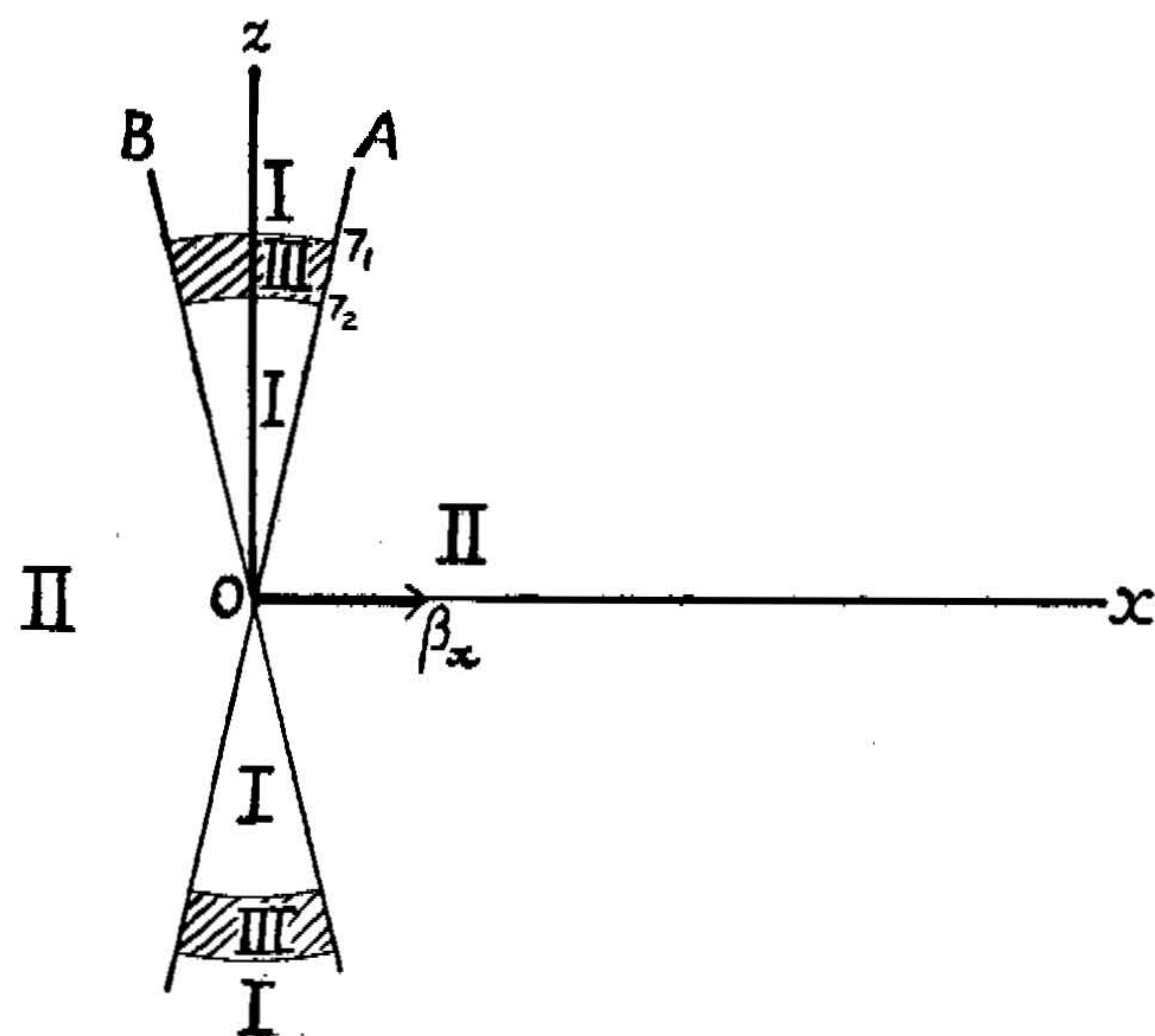
The simplest solutions of the equations (2') and (4') are the following: $\bar{\mathbf{M}}=0$ and $\bar{\mathbf{E}}=0$ for the whole space. Now $\bar{\mathbf{M}}=0$ gives $M_x=0$, $M_y = -\beta_x E_z$, and $M_z = \beta_x E_y$. Since $\bar{\mathbf{E}}=0$, we have $E_x=0$, $E_y(1-\beta_x^2)=0$, and $E_z(1-\beta_x^2)=0$. These equations can be satisfied by two different suppositions:

$$E_x=0, E_y=0, E_z=0; \text{ or } E_x=0, E_y \neq 0, E_z \neq 0, 1-\beta_x^2=0.$$

Let us divide the whole space round the point O into three parts:—Part I, annular space between two cones (fig. 2), obtained by rotation of OA and OB round the x-axis, having a common vertex at O, the angle AOB being infinitesimal. Part II, the inside of the above-mentioned cones. Part III is taken out of I by the spheres of r_1 and r_2 , with the centre

at O. For parts I and II we take the first solution of the equations (2') and (4'), i. e. $E_x=0$, $E_y=0$, $E_z=0$; hence $M_x=0$, $M_y=0$, $M_z=0$. For the region III we take the second solution: $E_x=0$, $E_y=-E_a \sin \alpha$, $E_z=E_a \cos \alpha$, $E_a \neq 0$, $\beta_x=+1$; hence $M_x=0$, $M_y=-E_a \cos \alpha$, $M_z=-E_a \sin \alpha$. Thus we obtain an electric field *only* in the region III. This field is moving with the velocity of light (first law of the light-quantum theory). If we transform the equation $\text{div } \mathbf{E}=0$, using polar coordinates r , ϕ , α , we obtain $\frac{\partial E_a}{\partial \alpha}=0$. Thus E_a is a function of r only; the cross-section of the Faraday tube is constant.

Fig. 2.



The equation $\text{div } \mathbf{M}=0$, which is also to be solved for the region III, after transformation takes the form

$$\frac{\partial}{\partial r}(r^2 M_r)=0 \quad \dots \quad (5)$$

or
$$\frac{\partial}{\partial r}(r^2 E_a)=0. \quad \dots \quad (5')$$

Here, namely in this equation (5), lies the divergence between the Maxwell-Hertz-Lorentz electrodynamics and the theory of Faraday-Maxwell-J. J. Thomson. From the standpoint of the first theory the equation (5) is valid in the whole space occupied by the free æther, and consequently will be valid in regions I and III and at their boundaries. So that if we admit $E_a=0$ and $M_r=0$ for the region I, the

constant in the solution $E_a = \frac{\text{const.}}{r^2}$ of the equation (5') for the region III must also vanish. So, according to the first theory, there can be no electromagnetic field in the region III, since there is no field in the regions I and II. We cannot admit the existence of a field in the region I, as

it must have the form $E_a = \frac{\text{const.}}{r^2}$, which gives for the point O, $E_a = \infty$.

From the point of view of Sir J. J. Thomson's theory the structure of the field is *discontinuous*. The field consists in a system of Faraday tubes moving relatively to one another; the intensity of the magnetic field is determined by the intensity of the electric field and by the normal velocity β_n of the tube, from the equation $\mathbf{M}=\beta_n \mathbf{E}$. It follows (1) that the equation (2') is satisfied identically; (2) that the intensity \mathbf{M} has a definite value different from zero only where the Faraday tubes are in motion. The equation (3), $\text{div } \mathbf{M}=0$, exists only where the Faraday tubes are in motion; in the rest of the free æther \mathbf{M} does not exist. If we admit the theory of J. J. Thomson, the equation $\text{div } \mathbf{M}=0$ must be satisfied in our case *only* in the region III up to its boundaries of II. At the boundary I-III we may have a discontinuity of M_r and consequently of E_a . Hence for the region III we may satisfy (5') by the supposition $E_a r^2 = \text{finite constant}$.

Calculating the energy of the ring, we obtain

$$\frac{1}{8\pi}(\mathbf{E}^2 + \mathbf{M}^2) \times \text{volume of the ring} = 2 \cdot \frac{1}{2} \epsilon \cdot E_a \cdot 2\pi r = \frac{2\pi^2 \epsilon^2}{c\phi_0} \cdot \nu,$$

where ϵ is the charge of the electron, $\frac{\nu}{c} = \frac{1}{2\pi r}$, and $\epsilon/2\phi_0$

the constant of integration. Thus we obtain the second law of the light-quantum theory (Law of Planck).

From the preceding considerations we may conclude that the Maxwell's equations actually lead to the solution given by Sir J. J. Thomson.

It seems to me also that the difference between the

Maxwell-Hertz-Lorentz electromagnetic theory and the theory of Faraday-Maxwell-J. J. Thomson might be *analytically* formulated as follows :—(1) For the free æther the differential equations are the same in both theories, being the Maxwell's system :

$$\operatorname{div} \mathbf{E} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\operatorname{curl} \mathbf{M} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\operatorname{div} \mathbf{M} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{and} \quad \operatorname{curl} \mathbf{E} = \frac{1}{c} \frac{\partial \mathbf{M}}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(2) The first theory postulated that all these equations are valid in the *whole* space filled by the free æther ; in the second theory the equation (3) occurs *only* where we have a moving electric field. Formulating this difference *geometrically*, we may say that the electric lines of force in the free æther are everywhere continuous, while the magnetic may be *discontinuous* : they must end on the boundaries dividing the moving Faraday tubes from those at rest. This circumstance gives the possibility of introducing *discontinuities* in electromagnetic field, and enormously increases the variability of the special instances, which satisfy the Maxwell systems of equations. The Maxwell equations, as it has been shown above, admit such forms of electromagnetic fields as have the same properties as the light-quanta.

It seems to me that this interpretation enables us to construct, on the basis of Maxwell's equations, investigating them for the case $\beta_n^2 = 1$, a *rational* quantum theory.

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